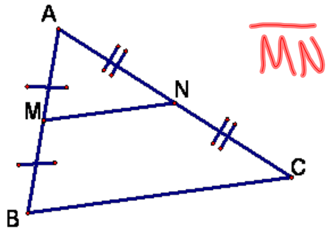
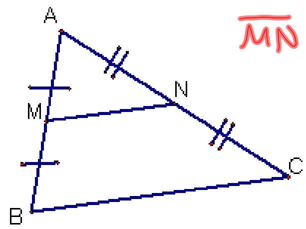
5.1 Midsegment Theorem and Coordinate Proof

Midsegment-of a triangle is a segment whose endpoints are the midpoints of two sides of a triangle.



$\overline{MN}$ is the midsegment

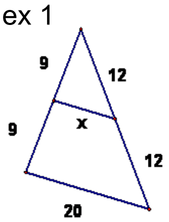
Theorem 5.1- Midsegment theoremA midsegment of a triangle is parallel to one side of the triangle, and its length is ½ the length of that side.



$\overline{MN} \parallel \overline{BC}$

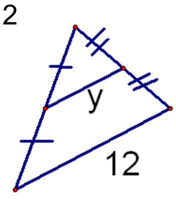
$MN = \frac{1}{2}BC$

ex 1



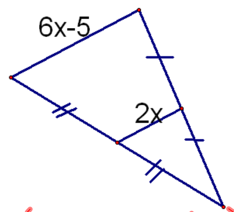
$x = 10$

ex 2



$y = 6$

ex 3



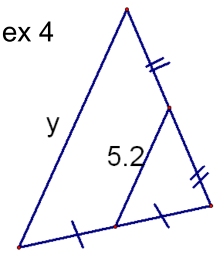
$6x - 5 = 2(2x)$

$6x - 5 = 4x$

$-5 = -2x$

$2.5 = x$

ex 4



$5.2 = \frac{1}{2}y$

$10.4 = y$

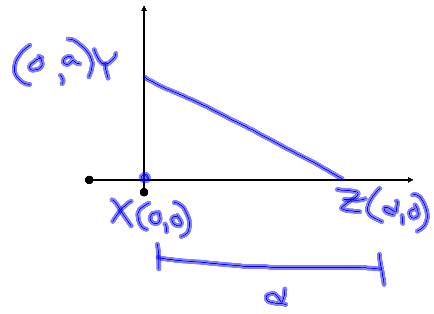
Coordinate Proof

Tips

1. Use Origin as vertex or center
2. At least one side on x-axis
3. 1st Quadrant if possible
4. Use easiest coordinates possible

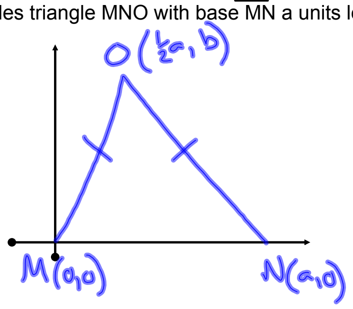
Example 5

Right triangle XYZ with hypotenuse $\overline{YZ}$
XZ = d units long

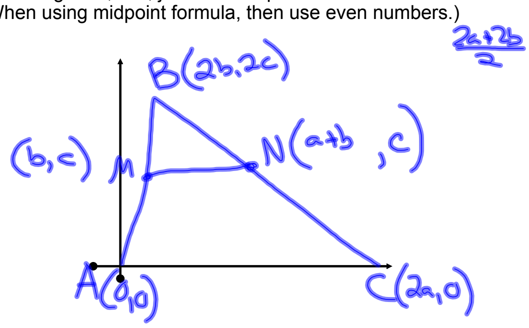


Example 6

Isosceles triangle MNO with base $\overline{MN}$ a units long

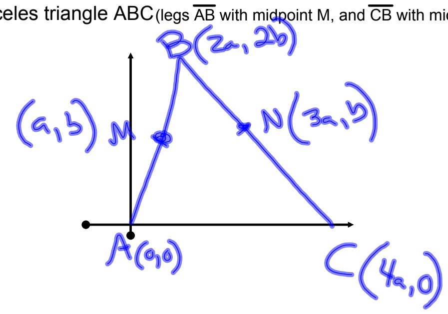


Example 7
A line segment, $\overline{MN}$, joins the midpoints of 2 sides of $\triangle ABC$
(When using midpoint formula, then use even numbers.)



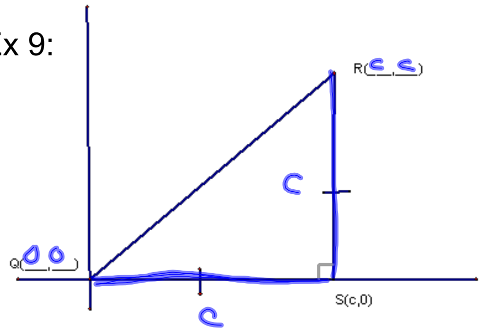
Example 8

Isosceles triangle ABC (legs $\overline{AB}$ with midpoint M, and $\overline{CB}$ with midpoint N)

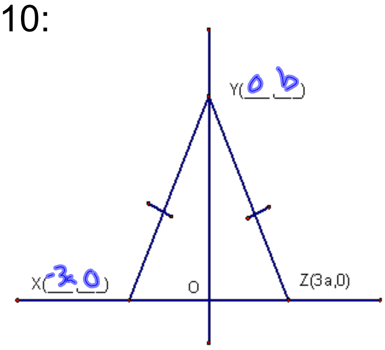


Fill in the missing coordinates.

Ex 9:



Ex 10:



Coordinate Proof

Distance Formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Slope $m=\frac{y_2-y_1}{x_2-x_1}$

Midpoint Formula

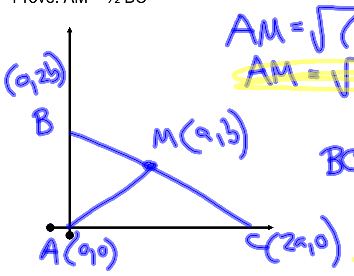
$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

When using midpoint formula, then use even numbers.

Example
Prove that the measure of the segment that joins the vertex of a right ∠ in a right △ to midpoint of the hypotenuse = ½ the measure of the hypotenuse

Given: Right△ABC with hypotenuse $\overline{BC}$. (M is the midpoint of $\overline{BC}$.)
Prove: AM = ½ BC

$$AM = \sqrt{(a-0)^2+(b-0)^2}$$
$$AM = \sqrt{a^2+b^2}$$



$$BC = \sqrt{(2a-0)^2+(0-2b)^2}$$
$$= \sqrt{4a^2+4b^2}$$
$$= \sqrt{4(a^2+b^2)}$$
$$BC = 2\sqrt{a^2+b^2}$$

$$\sqrt{a^2+b^2} = \frac{1}{2}\cdot 2\sqrt{a^2+b^2}$$
$$\sqrt{a^2+b^2} = \sqrt{a^2+b^2} \checkmark$$

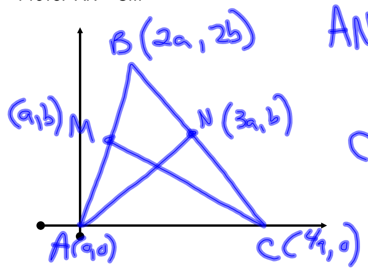
$$\therefore \; AM = \frac{1}{2}BC$$

therefore

The segments joining the vertices to the midpoints of the legs of an isosceles triangle are congruent.

Given: Isosceles triangle ABC (legs $\overline{AB}$ with midpoint M, and $\overline{CB}$ with midpoint N)

Prove: AN = CM



AN =

CM =

HW
Use example 7 from notes to prove the midsegment theorem and
p298-299 #s 3-5, 15-19, 24